Frequency Enhancement of Seismic Data via Tunable Q-factor Wavelet Transform

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GeoConvention 2012:Vision

Summary

We present a technique to enhance the frequency spectrum of seismic data via the tunable Q-factor wavelet transform. The proposed method increases the bandwidth of the data while maintaining the phase and temporal position of events, thus minimizing high frequency noise and artifact contamination. The ability of the tunable Q-factor wavelet transform (TQWT) to sparsely represent data along with the capability to manipulate the central frequency to bandwidth ratio of the wavelet basis used for decomposition, are beneficial attributes for the accurate preservation of event information after the frequency enhancement. The effectiveness of the technique is examined by applying it to real seismic data and analyzing the temporal and spectral results.

Introduction

A seismic trace can be defined as the convolution of two unknown discrete time series: a source wavelet, and a reflectivity series. The reflectivity series can be thought of as a representation of our unknown geology; a series of amplitude varying delta functions that delineates the amount of source energy reflected off subsurface features at different depths in time (Ulrych and Sacchi, 2005). In exploration seismology it is impossible to produce a broadband source wavelet (a discrete delta function), thus a recorded seismic trace will always be band-limited by the source wavelet. Due to the trade off between temporal and spectral resolutions, the extent of band-limitation on a seismic trace determines the resolution of the smallest possible subsurface features. Consequentially, the under-constrained problem arises of how the band-limited frequency content of a seismic trace can be enhanced and a more accurate estimation of the reflectivity series recovered.

Many deconvolution techniques such as predictive deconvolution, homomorphic deconvolution, Kalman filtering, and deterministic deconvolution approach this problem by attempting to separate the source wavelet from the seismic trace (Arya and Holden, 1978). However, more recently wavelet transform techniques such as thin-bed reflectivity inversion and bandwidth extension have been becoming a more popular approach to this problem (Chopra et al., 2006; Smith et al., 2008; Zhang and Castagna, 2011). The sparsity attributes of the wavelet transform, combined with the capacity for analyzing time-frequency information simultaneously within different frequency subbands or temporal resolutions, makes the wavelet domain ideal for the given problem. We present a frequency enhancement technique that operates within the tunable Q-factor wavelet domain by increasing energies of the frequency subbands dampened by the source wavelet, using the dominant frequency subband information required to sparsely represent the seismic trace.

Theory

Recently, Selesnick (2011) introduced a multi-resolution analysis toolbox for which the quality factor or Q-factor is easily specified. The Q-factor of a given wavelet is a measure of the wavelets central frequency to bandwidth ratio, and controls the oscillatory behavior of the wavelet. The tunable Q-factor wavelet transform is a discrete-time wavelet transform, in which the usual wavelet variables of position (time) and scale (frequency subband) are considered, along with an additional variable of Q-factor (central frequency to bandwidth ratio of wavelets). A subsequent two-channel filter bank algorithm implements the TQWT. Using scaling operators in frequency to implement the subsequent high and low pass filters; the TQWT can be implemented quickly and efficiently via the Radix 2-FFT. In addition, the TQWT software package comes with built in sparsity functions allowing sparse fittings of data in the TQWT domain via the iterative split augmented lagrangian shrinkage algorithm (SALSA) (Selesnick, 2011). The Sparse representations of data within the TQWT domain will be important in the presented frequency enhancement technique, as the reflectivity series is assumed to satisfy the sparse condition. Thus, sparse representation within TQWT domain will delineate an estimate of reflective energies or reflective supports within a seismic trace in both time and frequency simultaneously.

The following notation will be used in describing the method. $\mathbf{Q}$ is the tunable Q-factor wavelet transform operator, and $\mathbf{Q}^{\mathrm{T}}$ its adjoint operator. When $\mathbf{Q}$ is applied to the data $\mathbf{d}$, it decomposes into a set of wavelet coefficients $\mathbf{W}_{\mathrm{n}}$ consisting of N subbands with differing temporal and spectral resolutions.

$$\mathbf{Qd} = \mathbf{W}_{\mathrm{n}} \tag{1}$$

$$\mathbf{W}_{\mathrm{n}} = [\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3 ... \mathbf{W}_{\mathrm{N}}] \tag{2}$$

Applying the adjoint operator $\mathbf{Q}^{\mathrm{T}}$ to the wavelet coefficients $\mathbf{W}_{\mathrm{n}}$ perfectly reconstructs the data $\mathbf{d}$.

$$\mathbf{Q}^{\mathrm{T}}\mathbf{W}_{\mathrm{n}} = \mathbf{d} \tag{3}$$

A Sparse set of wavelet coefficients can be found by minimizing the cost function $\mathbf{J}$ via the iterative SALSA algorithm, where λ is the tradeoff parameter of the cost function.

$$\mathbf{J} = \left\| \mathbf{d} - \mathbf{Q}^{\mathrm{T}}\mathbf{W}_{\mathrm{n}} \right\|_2^2 + \lambda \left\| \mathbf{W}_{\mathrm{n}} \right\|_1 \tag{4}$$

The sparse set of wavelet coefficients localizes the energies to fewer positions on fewer subbands and represents the dominant frequency subband information required to sparsely represent the seismic trace. This information gives an estimate of where reflective energies or reflective supports of the seismic trace are located in time and frequency simultaneously. However, due to the payoff between spectral and temporal resolutions there is uncertainty in the exact time, and frequency of where these reflectivity supports are located. Using these sparse coefficients, a masking function M is created that estimates the reflectivity supports of the signal, and is scaled to N different temporal/spectral resolutions to produce $\mathbf{M}_{\mathrm{n}}$. It is important that the Q-factor of the wavelet basis used to decompose the data best places the dominant energies of the seismic trace within the middle subbands, providing the optimal payoff between temporal and spectral resolutions. This makes the rescaling of the masking function less intrusive, helping better preserve the relative amplitudes of events and reducing artifacts in the frequency enhanced data. The masking functions $\mathbf{M}_{\mathrm{n}}$ are then multiplied to the original wavelet coefficients $\mathbf{W}_{\mathrm{n}}$ producing a set of wavelet coefficients masked by the scaled estimates of reflectivity support $\mathbf{W}'_{\mathrm{n}}$.

$$\mathbf{W}'_{\mathrm{n}} = [\mathbf{W}_1\mathbf{M}_1, \mathbf{W}_2\mathbf{M}_2, \mathbf{W}_3\mathbf{M}_3 ... \mathbf{W}_{\mathrm{N}}\mathbf{M}_{\mathrm{N}}] \tag{5}$$

Subband energy of the masked wavelet coefficients is then normalized by the operator $\mathbf{N}$ generating a set of wavelet coefficients $\mathbf{W}''_{\mathrm{n}}$ with near equal energy in each subband (except the Nyquist subband). It is important to note that the placement of all energy within $\mathbf{W}''_{\mathrm{n}}$ is from the original decomposition by $\mathbf{Q}$. This ensures that the phases and positions of events are well maintained.

$$\mathbf{N}\mathbf{W}'_{\mathrm{n}} = \mathbf{W}''_{\mathrm{n}} \tag{6}$$

The adjoint operator $\mathbf{Q}^{\mathrm{T}}$ is then applied to $\mathbf{W}''_{\mathrm{n}}$ to reconstruct a new data set with an enhanced frequency spectrum and improved temporal resolution $\mathbf{D}$.

$$\mathbf{Q}^{\mathrm{T}}\mathbf{W}''_{\mathrm{n}} = \mathbf{D} \tag{7}$$

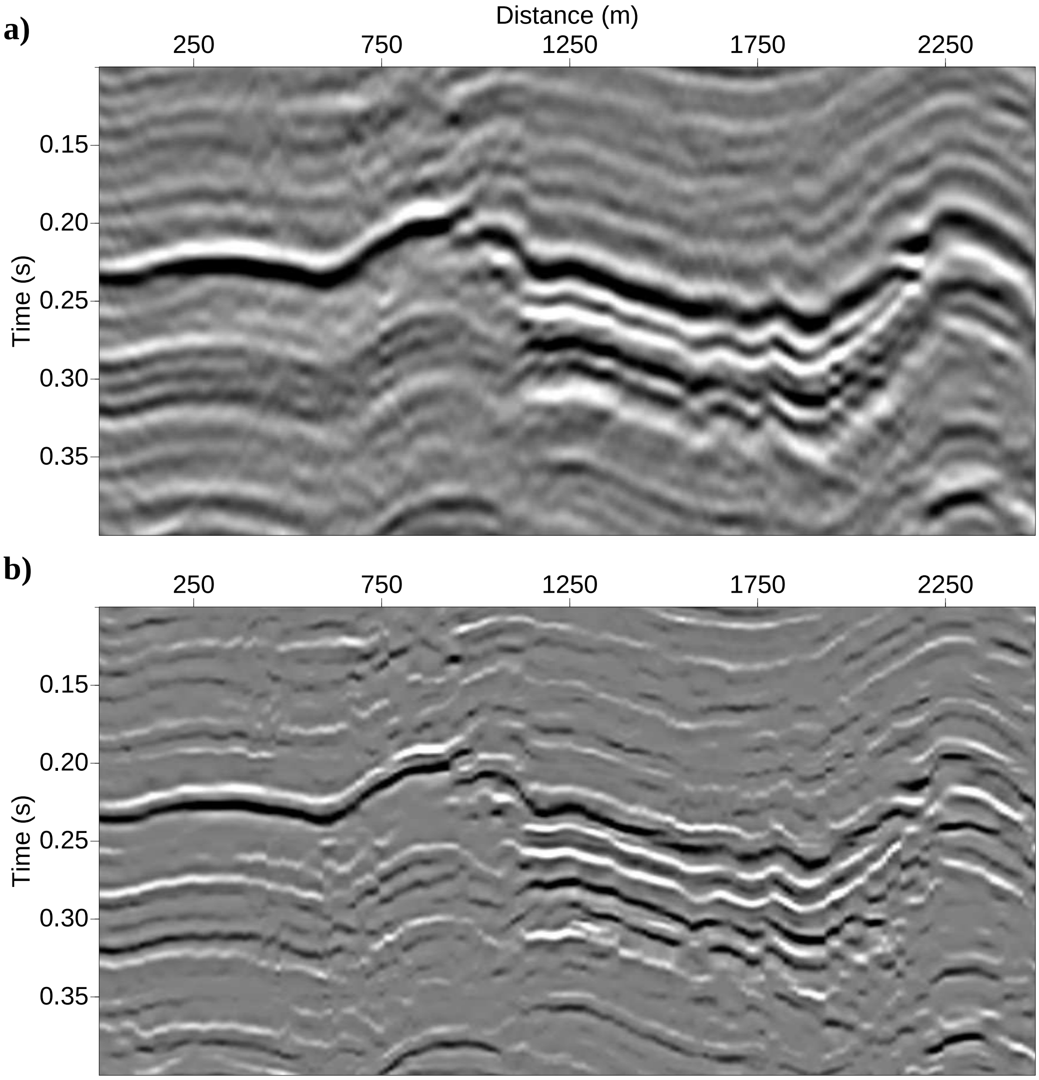

Figure 1 *: a) Original stacked seismic data. b) Frequency enhanced stacked seismic data.*

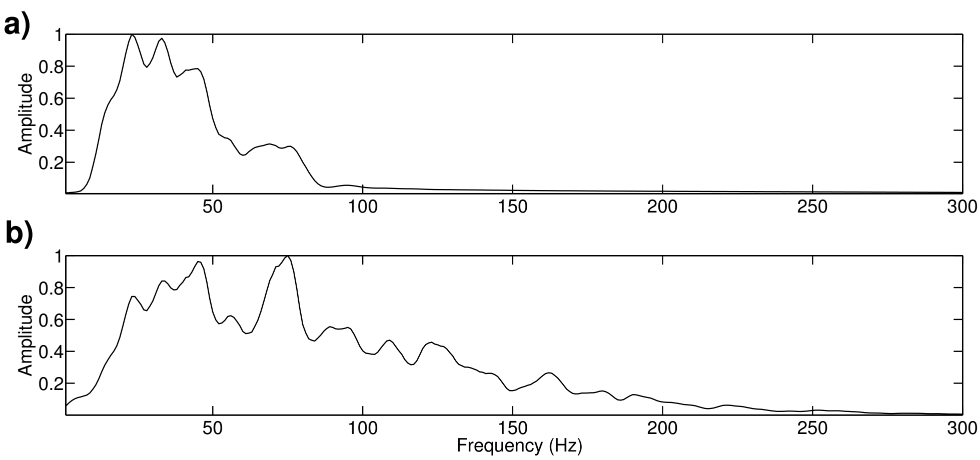

Figure 2 *: a) Normalized spectrum of trace within original stacked seismic data. b) Normalized spectrum of same trace within frequency enhanced stacked seismic data.*

Examples

Figure 1b shows the results of applying the propose method to stacked seismic data in Figure 1a with a sampling rate of one millisecond. It can be seen that the phases and temporal positions of events are well maintained, along with the relative amplitudes of events. Also shown is the normalized frequency spectrum of a single trace within the original data Figure 2a, and the normalized frequency spectrum for the same trace within the frequency enhanced data Figure 2b. It can be seen from the bandwidth of the frequency enhanced trace in Figure 2b in comparison to that of Figure 2a, that the temporal resolution is improved by approximately an octave without contaminating Figure 1b with high frequency noise or substantial artifacts.

Conclusion

Sparse fittings and the ability to analyze time and frequency information simultaneously within wavelet domain, make it optimal for the frequency enhancement of band-limited seismic data. we present a frequency enhancement technique that uses the tunable Q-factor wavelet transform. The tunable Q-factor wavelet transforms sparse fittings and its ability to easily manipulate the Q-factor of the wavelet basis used in decomposition are beneficial attributes in accurately preserving event information after the frequency enhancement. The effectiveness of the method is shown applying it to real seismic data and analyzing the temporal and spectral results. It is observed that the technique can effectively enhance the frequency content of seismic data, improving temporal resolution by approximately an octave without the contamination of high frequency noise or substantial artifacts. Thus preserving the phase and temporal position of events, along with reasonable relative amplitudes.

Acknowledgements

The authors are grateful to the sponsors of Signal Analysis and Imaging Group (SAIG) at the University of Alberta.

R. Ferner would like to thank David Bonar for some interesting discussions about multi-resolution analysis.

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